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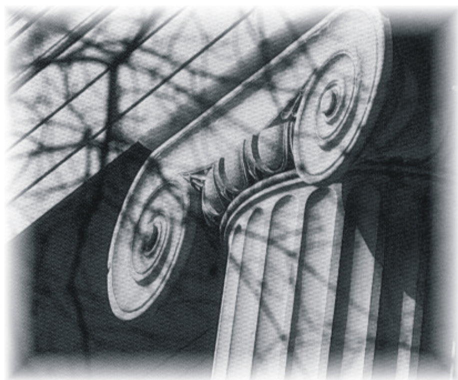

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LOTTERIES, PUBLIC GOOD PROVISION AND THE DEGREE OF RIVALRY

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Abstract

Under the standard summation technology, pure public goods can be provided via the direct contributions mechanism, even in an arbitrarily large group. However, if the public good exhibits any degree of rivalry, individual consumption of the public good will fall to zero as group size grows large. Thus, the direct contributions mechanism is not robust to the introduction of rivalry. By contrast, Morgan's (2000) lottery mechanism is robust to the introduction of rivalry when the lottery prize is proportional to group size. The lottery mechanism can provide public goods in a large group when the public good exhibits a degree of rivalry, provided that the degree of rivalry is not too high. This suggests that the lottery mechanism can provide a broader range of public goods in a large group than the direct contributions mechanism.

JEL Codes: D7, H4, C72

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1. INTRODUCTION

Morgan (2000) has shown that provision of a *pure* public good is greater under the lottery mechanism than under the direct contributions mechanism. Moreover, under the lottery mechanism, provision is increasing in group size. However, Pecorino and Temimi (2007) consider *fully rival* public goods, and while they find that the lottery mechanism continues to outperform the direct contributions mechanism, under both mechanisms, individual consumption of the public good goes to 0 in a large group. While Pecorino and Temimi consider the extreme cases of a pure public good and a fully rival public good, they do not consider cases with intermediate degrees of rivalry. These intermediate cases are important because if, for example, the introduction of even a small degree of rivalry causes a mechanism to fail in a large group, then it suggests a rather limited scope for the mechanism's usefulness. Indeed, for the voluntary contributions mechanism, we find that the introduction of a small degree of rivalry implies that consumption of the public good will fall to 0 in a large group. By contrast, the lottery mechanism continues to perform well in large groups, as long as the lottery prize increases in proportion to group size and the degree of rivalry of the public good is not too high. The robustness of the lottery mechanism with respect to the introduction of rivalry suggests a potentially large range of public goods for which the mechanism might be effective.

Much work on public goods has focused on the case of pure public goods, but the works of Borcherding and Deacon (1972) and Bergstrom and Goodman (1973) indicate that publically provided goods exhibit a great deal of rivalry.[1] Thus, in assessing the overall usefulness of the lottery mechanism, it is useful to see how well it performs when a degree of rivalry is introduced.

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[1] For example, it should be clear that police protection exhibits a high degree of rivalry in the following sense: If we hold the number of police officers constant, then the effective police protection enjoyed by an individual will decrease as the number of people in the police jurisdiction increases.

2. THE MODEL

We will first establish that the voluntary contributions mechanism is not robust to rivalry in the sense that introducing a small degree of rivalry will guarantee that individual consumption of the public good will fall to zero as group size grows large. The exposition of the model follows Pecorino and Temimi (2007) fairly closely and this work in turn draws on Morgan (2000). The key difference with these earlier works is that we will allow the public good to exhibit intermediate degrees of rivalry.

2.1 The voluntary contributions mechanism

There are *n* identical consumers with utility given by

$$U_i = w - x_i + h(g_i)\,, \tag{1}$$

where *w* denotes the wealth of the consumer, *h(g)* reflects utility derived from the public good, x_i is person *i*'s contribution to the public good and g_i is the individual consumption of the public good. Since the public good is nonexcludable, we will write $g_i = g$ throughout. We assume that $h'(.) > 0$, and $h''(.) < 0$, so that *h(g)* is strictly concave. In addition, we assume that $h'(\infty) = 0$, $h'(y)$ is finite for y > *0*, and $h'(0) > 1$. The last assumption ensures that provision of the public good is socially desirable, even when it is fully rival.

Per person consumption of the public good is given by

$$g = \frac{\sum_{j=1}^{n} x_j}{n^{\beta}}\,. \tag{2}$$

The parameter β reflects the degree of rivalry. If β = 0, we have a pure public good, and if β =1, we have a fully rival public good. If 0 < β < 1, then we have an intermediate case.

Under the voluntary contributions mechanism, consumers maximize (1) subject to (2). At an interior equilibrium, the first order condition implies

$$h'(g) = n^{\beta}. \quad (3)$$

If $\beta = 0$, we have a pure public good and provision is invariant to group size. This is the traditional result under quasi-linear preferences. Among other things, this implies that a pure public good can be provided even as the group size grows large. If $\beta > 0$, then g must go to zero in a large group. The reason is that n^{β} rises to infinity, but $h'(g)$ is finite for $g > 0$. Thus, the condition in (3) cannot hold for large n unless g approaches 0. This is summarized as follows:

> Result 1: Under the direct contributions mechanism, if there is any degree of rivalry, per person consumption of the public good must go to 0 as n approaches ∞.

The key point is that the voluntary contributions mechanism is not robust to the introduction of rivalry in the sense that the mechanism will totally fail to provide positive levels of g in a sufficiently large group if the public good exhibits any degree of rivalry. Pecorino (2008) shows that this result holds for a more general utility function which allows for the possibility of income effects on the demand for the public good.

2.2. The Lottery Mechanism

While Pecorino and Temimi (2007) consider the extreme cases of a pure public good and a fully rival good, here we consider the intermediate cases. This will allow us to see if the lottery mechanism is robust to the introduction of rivalry in the sense that the mechanism will still function in large groups when a small degree of rivalry is introduced.

The utility function is the same as in (1) except now the contribution x_i gives the individual a chance to win a lottery. Individual consumption of the public good is the difference between the sum of the contributions and the lottery prize R divided by n^{β}, where β reflects the degree of rivalry:

$$g = \frac{\sum_{j=1}^{n} x_j - R}{n^{\beta}}. \tag{4}$$

The probability that individual i wins the lottery is simply $x_i / \sum_{j=1}^{n} x_j$. Individuals choose x_i to maximize

$$U_i = w - x_i + \frac{x_i}{\sum_{j=1}^{n} x_j} R + h(g), \tag{5}$$

subject to (4). An interior equilibrium can be guaranteed if we assume, as in Morgan (2000: 767), that the group putting on the lottery has access to a small amount of deficit financing δ. In addition, the symmetry of the players guarantees that this interior equilibrium will be symmetric. Thus, after taking the first order condition, we can impose $x_j = x \ \forall j$ to yield

$$\frac{(n-1)R}{n^2 x} + (1/n^{\beta}) h'\left(\frac{nx - R}{n^{\beta}}\right) = 1. \tag{6}$$

Morgan (2000) has shown that the lottery provides a greater level of the public good than the direct contributions mechanism. It is straightforward to show that this continues to be true, regardless of the degree of rivalry.[2]

As in Pecorino and Temimi (2007), we will consider two options for how the lottery prize *R* evolves as a function of group size. First, we will assume that the prize is constant and then we will assume that the prize increases in proportion to group size. This is summarized in (7):

$$R = \overline{R} \tag{7a}$$

$$R = nr \tag{7b}$$

Note that the lottery prize cannot grow faster than the group size *n*. If it did, at some point we would have $R - nw > 0$, where nw is the maximum aggregate bid. If $R > nw$, we cannot have a positive level of provision of the public good.

First consider $R = \overline{R}$ and β > 0. As group size grows large, the condition in (6) is approximately

$$\frac{\overline{R}}{nx} + (1/n^{\beta})h'\left(n^{1-\beta}x\right) = 1. \tag{8}$$

To see that *g* must approach 0 in a large group, we can assume that it is positive and then show that this must violate the condition in (8). Per person consumption of the public good is approximately $n^{1-\beta}x$. If this is strictly positive, $h'\left(n^{1-\beta}x\right)$ is finite and $(1/n^{\beta})h'\left(n^{1-\beta}x\right) \rightarrow 0$ as $n \rightarrow \infty$. Moreover, if $n^{1-\beta}x$ is strictly positive, then $nx \rightarrow \infty$ as *n* grows large. This implies that

[2] When $R = 0$, the condition in (6) is the same as in (3) and the lottery mechanism provides the same level of provision as direct contributions. Provision under the lottery is increasing in *R*, so if the lottery prize is increased above 0, provision under the lottery will exceed that under the direct contributions mechanism.

$\overline{R}/nx$ approaches 0. Since both terms on the left-hand side approach 0, the condition in (8) is violated for large n whenever $g > 0$. Thus, g must approach 0 as group size grows large. This is summarized as follows:

> Result 2: Under the lottery mechanism with a constant prize $\overline{R}$, per person consumption of the public good g approaches 0 as group size grows large whenever the public good exhibits a positive degree of rivalry $\beta > 0$.

As with the direct contributions mechanism, if the prize is constant, the lottery mechanism is not robust to the introduction of a small degree of rivalry.

Next, consider what happens when the lottery prize rises in proportion to group size so that $R = rn$ and $\beta > 0$. Equation (6) becomes

$$\frac{(n-1)r}{nx} + (1/n^{\beta})h'\left(n^{1-\beta}(x-r)\right) = 1 . \tag{9}$$

Individual consumption of the public good is $g = n^{1-\beta}(x-r)$. Assuming that g is strictly positive as n grows large does not imply a violation of the first order condition. This stands in contrast to the direct contributions mechanism and the lottery mechanism when the prize is constant. This does not necessarily imply that provision is bounded away from 0 as $n \to \infty$, only that this is a possibility. Ultimately, the value of g in a large group will depend upon the degree of rivalry (i.e., β) and the shape of the $h(g)$ function. To illustrate this, let $h(g) = ln(g)$. From equation (9), we can derive the following solution for g:

$$g = \frac{1-r}{2n^{\beta}} + \sqrt{\frac{(1-r)^2}{n^{2\beta}} + \frac{nr}{n^{2\beta}}} . \tag{10}$$

If $\beta > ½$, g falls to 0 in a large group, but if $\beta < ½$, g rises without bound as n grows large. For this example, $\beta = ½$ divides the parameter space into an area where the lottery ceases to function well in a large group, and an area where it continues to function well. In general, this cutoff value of β will depend on the specific functional form for h. What this example illustrates, however, is that the introduction of a small degree of rivalry does not imply that g falls to 0 in a large group. When the lottery prize is increased in proportion to group size, the lottery mechanism is robust to the introduction of a small degree of rivalry. In addition, for $\beta < ½$, the lottery mechanism performs quite well in a large group in the sense that g rises without bound in n. This is summarized as follows:

> Result 3: Under the lottery mechanism when $R = rn$, the per person consumption of the public good need not fall to 0 in a large group in the presence of rivalry. When $h(g) = ln(g)$, g falls to 0 in a large group if the degree of rivalry $\beta > ½$, but rises to ∞ if $\beta < ½$.

This result is significant, because it suggests that the lottery mechanism is potentially effective over a broader range of goods than the voluntary contributions mechanism. As noted earlier, many publically provided goods exhibit at least some degree of rivalry. Thus, if a mechanism is unable to provide any public good to a large group in the face of even a small degree of rivalry, it suggests that the mechanism will be ineffective over a broad range of goods.

If we assume that $h(g) = g^{\alpha}$, we cannot obtain analytical solutions for the model. However, numerical simulations support the results found with the natural log function. In particular, when the degree of rivalry is positive, but small, *g* is found to increase in group size, but when β is sufficiently high, *g* decreases in group size and approaches 0 as *n* grows large.[3]

3. Conclusion

Previous work has shown that the lottery mechanism is effective in providing pure public goods, but that the mechanism breaks down in large groups when the good is fully rival. In this paper, we show that the effectiveness of the lottery mechanism is not knife edged. That is, the introduction of a small degree of rivalry to the public good will not, in general, lead to a breakdown of the mechanism in large groups if the lottery prize is increased in proportion to group size. By contrast, the voluntary contributions mechanism totally breaks down in a large group when the public good exhibits even a small degree of rivalry.

While the results of this paper are generally favorable for the lottery mechanism, it should be noted that if the degree of rivalry is sufficiently high, that the mechanism will break down in a sufficiently large group. In addition other factors, such as the distributional effects of the lottery, may mitigate against the widespread use of the lottery mechanism to finance public goods.

[3] For the experiments we ran, we found that the breakpoint for β generally stayed in the neighborhood of ½, as we varied the value of α.

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