

FOR PRACTICE ONLY -- 2011

University of Alabama

Economics 610
Advanced Microeconomic Theory

Fall 2010
Professor Schlesinger

Exam 1

Answer all 5 questions below. Be sure to explain how you derive your answers.

1. State whether each of the following statement is TRUE or FALSE and provide your reasons for saying so. All of the credit for this question is based upon your explanations.

- (a) A Cournot-Nash equilibrium in a duopoly market is Pareto efficient; i.e., it is not possible for both firms to earn a higher profit.
- (b) The profit function is homothetic in prices.
- (c) The cost function $c(y, w_1, w_2)$ is homogeneous of degree 0 in (y, w_1, w_2) , where y indicates the firm's output and w_i is the price of input i , $i = 1, 2$.
- (d) A production function that exhibits increasing returns to scale must be homogeneous of some degree k , where $k > 1$.

2. A competitive market consists of three firms with the following cost functions:

$$c_1(y_1) = y_1^2 + 2, \quad c_2(y_2) = y_2^2 + 3, \quad \text{and} \quad c_3(y_3) = y_3^2 + 4.$$

Any potential new firms that enter the market will have a cost function of $c_i(y_i) = y_i^2 + 5$. The market demand is given as $Y = 10 - p$.

- (a) Find the (short-run) aggregate supply function for this market.
- (b) Find the (short-run) equilibrium price.
- (c) Could this short run equilibrium price also be the long-run equilibrium price? Explain why or why not.

3. A market has (inverse) demand given by $P = 100 - y$. There are two firms producing identical products, with cost functions $c_1(y_1) = 15y_1$ and $c_2(y_2) = \frac{1}{2}y_2^2$.

Determine the equilibrium **price** and the **quantity** produced by each firm for each of the following types of markets. Be sure to explain your answers.

- (a) A competitive market, with both firms acting as price takers.
- (b) A cartel that maximizes joint profit.
- (c) A Cournot-Nash duopoly.

4. (a) Consider the production function $f(x) = 1000x - x^2$ for $x < 500$. Assume that the firm cannot use more than 500 units of input due to prohibitive additional costs. We also assume that $w < 1000p$. [If this last inequality does not hold, it can be shown that the firm would shut down in the short run. You do not need to show this.] Find the factor demand function $x(p, w)$ and the supply function $y(p, w)$, and show that both are homogeneous in prices.

(b) Consider the profit function $\pi(p, w) = 500(500p - w) + \frac{w^2}{4p}$. Use Hotelling's Lemma to find the factor demand function $x(p, w)$ and the supply function $y(p, w)$.

5. (a) A firm has the production function $f(x_1, x_2, x_3, x_4) = \sqrt{\min(x_1 + x_2, x_3 + x_4)}$. Find the firm's cost function $c(y, w_1, w_2, w_3, w_4)$.

(b) A firm has the cost function $c(y, w_1, w_2) = w_1y + w_2y^2$. Use Shephard's Lemma to find the firm's production function $f(x_1, x_2)$

Exam 2

Answer all 5 questions below. Be sure to explain how you derive your answers.

1. State whether each of the following statement is TRUE or FALSE and provide your reasons for saying so. All of the credit for this question is based upon your explanations.

- (a) $e(v(m, p_1, p_2), p_1, p_2) = m$, where e denotes the expenditure function and v denotes the indirect utility function.
- (b) A homothetic utility function must be concave.
- (c) An exchange equilibrium will always exist, even if preferences are not convex. (Recall: convex preferences are defined as the upper contour set being convex.)
- (d) For utility defined over two goods, the indirect utility function is homogeneous of degree 0 in prices (p_1, p_2) , assuming that the level of income m is held constant.

2. Consider the model of the Stolper-Samuelson Theorem. [We have two inputs, producing two goods: Food and Drink. Technologies are strictly convex with constant returns to scale. Food is input-1 intensive. World markets are competitive with prices p_F and p_D for Food and Drink respectively.] Suppose that the price of drink p_D decreases by 1%, but that p_F does not change. Which of the following effects are possible in equilibrium? All credit for this problem is for your explanation as to why each effect is possible or not possible.

- (a) w_1 increases?
- (b) w_2 increases?
- (c) w_2 decreases by less than 1% ?
- (d) more food is produced?

3. AG (the “accounting guy”) has a monthly budget of m dollars to spend on good 1 and good 2 (both are “normal goods”). Last month, AG purchased the bundle: (\bar{x}_1, \bar{x}_2) , where $\bar{x}_1 > 1$, $\bar{x}_2 > 0$. This month, AG’s choices and preferences are exactly the same as last month, but a sale on good 1 has been altered. Last month, the price of good 1 was p_1 per unit. This month, the price is still p_1 for the first unit of good 1 purchased, but the price is kp_1 per unit for any additional good 1 purchased, where $k > 1$. The price of good 2, p_2 , is unchanged since last month.

- (a) Describe this month’s budget constraint for AG.
- (b) Will AG necessarily buy less good 1 this month than last month? Explain. (To receive full credit for this question, you must show this solution analytically. If you cannot do this, you will receive partial credit for a neat graphical solution with an explanation of your graph.)

4. There are two consumers in an exchange economy with the following preferences and endowments:

$$u^A(x_1, x_2) = x_1^2 x_2^3, \quad \bar{\omega}^A = (1, 0)$$

and

$$u^B(x_1, x_2) = \min(2x_1, 2x_2), \quad \bar{\omega}^B = (0, 1).$$

- (a) Find all Pareto efficient allocations in this economy.
- (b) Find the Walrasian Equilibrium allocation and prices.
5. (a) Consider the indirect utility function $v(m, p_1, p_2) = \frac{m}{p_1 + p_2}$. Find the direct utility function, $u(x_1, x_2)$.
- (b) Consider the expenditure function $e(u_0, p_1, p_2) = (p_1 + p_2)u_0$. Find the indirect utility function $v(m, p_1, p_2)$.
- (c) Consider the direct utility function $u(x_1, x_2) = \min(2x_1 + x_2, x_1 + 2x_2)$. Find the indirect utility function $v(m, p_1, p_2)$.