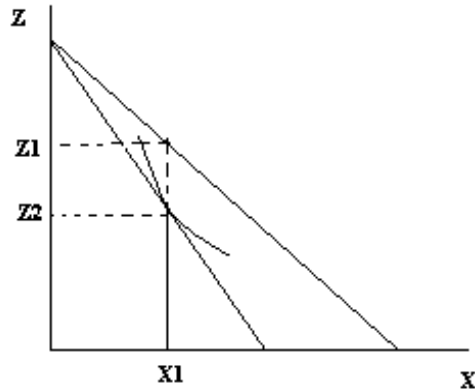


Problem Set II



1. Consider the picture above:
The point (X_1, Z_1) is on the original constraint:

$$I = pX + Z$$

The point (X_1, Z_2) is on the with-tax constraint:

$$I = (p + t)X + Z$$

Prove that: $Z_1 - Z_2 = tX_1$

So, the tax paid in the with-tax equilibrium is indicated by the vertical distance between the constraints at the equilibrium point.

2. We defined the compensating variation as:

$$CV^{oi} \text{ solves } V(p^i, I^i - CV^{oi}) = V(p^o, I^o)$$

Derive:

$$CV^{oi} = I^i - E(p^i, U^o)$$

3. We have a single individual who consumes two goods and labor with utility function:

$$U(X_1, X_2, L) = a_1 \frac{X_1^{\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)}}{1 - \frac{1}{\varepsilon_1}} + a_2 \frac{X_2^{\left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)}}{1 - \frac{1}{\varepsilon_2}} - sL, \quad 0 < \varepsilon_1 < 1, 0 < \varepsilon_2 < 1$$

She faces budget constraint

$$q_1 X_1 + q_2 X_2 = wL + I$$

We also stipulate labor is the numeraire ($w=1$) and there is no exogenous income ($I=0$). However, you will want to use duality results to ease the work, so do not substitute in the specific values.

- A) Show that the Marshallian demand for good 1 is:

$$\frac{d \ln X_1}{d \ln q_1} = -\varepsilon_1$$

- B) Show that the indirect utility function is:

$$V(w, q_1, q_2; I) = \left(\frac{a_1}{\varepsilon_1 - 1} \right) \left(\frac{a_1 w}{q_1 s} \right)^{\varepsilon_1 - 1} + \left(\frac{a_2}{\varepsilon_2 - 1} \right) \left(\frac{a_2 w}{q_2 s} \right)^{\varepsilon_2 - 1} + \frac{sI}{w}$$

- C) Show that the expenditure function is:

$$e(w, q_1, q_2; U) = \frac{w}{s} U - \frac{w}{s} \left(\frac{a_1}{\varepsilon_1 - 1} \right) \left(\frac{a_1 w}{q_1 s} \right)^{\varepsilon_1 - 1} - \frac{w}{s} \left(\frac{a_2}{\varepsilon_2 - 1} \right) \left(\frac{a_2 w}{q_2 s} \right)^{\varepsilon_2 - 1}$$

- D) Derive the full (3x3) Slutsky matrix.
 E) Use your result from (B) to derive the optimal unit-tax structure by utility maximization assuming constant opportunity costs.

As a matter of notation, the government's constraint is:

$$t_1 X_1^* + t_2 X_2^* = R \quad \text{and} \quad q_1 = p_1^0 + t_1, \quad q_2 = p_2^0 + t_2$$

with no dependence of the p 's on the t 's. Your answer should be something like:

$$\frac{t_1}{q_1} = \frac{\varepsilon_2}{\varepsilon_1} \frac{t_2}{q_2}$$

- F) Use the discussion of the Ramsey rule in the notes to provide as complete an interpretation as possible of your result in (E).
4. Suppose the demand for good X can be represented by the following equation: $X_d = 36 - 4P$. Furthermore, suppose that the demand for good Y can be represented by: $Y_d = 29 - P$.
- A) Find the elasticity of demand for both good X and good Y when the price is \$6.
 B) Suppose that an Ad Valorem tax is placed on both goods. Good Y is taxed at a rate of 4%. To insure that the *inverse elasticity rule* holds, what must be the rate at which good X is taxed?
 C) At a price of \$8, what must the tax rate for good X be if the tax rate on good Y remains at 4%?

Reminder: Elasticity at a given price is found using the formula: $\varepsilon = -(1/S)(P/X)$ where S is the slope of the demand curve, X is the quantity demanded, and P is the price.

5. Suppose that an individual has a utility function $U(X, Y) = 3X^{1/3}Y^2$. If the price of X is 4, the price of Y is 5, and income is 40, find the optimal bundle of X and Y.
- A) Suppose that the price of X increases to 4.5 due to a tax. What is the amount of equivalent variation associated with this tax?
 B) What is the excess burden associated with this tax?

6. Consider a consumer with indirect utility function $V = a - \frac{q_1^{1-\varepsilon_1}}{[1-\varepsilon_1]w^{1-\varepsilon_1}} - \frac{q_2^{1-\varepsilon_2}}{[1-\varepsilon_2]w^{1-\varepsilon_2}} + \frac{I}{w}$.
 $0 < \varepsilon_i < 1, i = 1, 2$

where I is lump-sum income, q_1 and q_2 the prices of the two available goods and w the wage rate.

- i. Selecting w as the numeraire with unit price, so that the marginal utility of income is equal to 1, use Roy's identity to show the demand for good i is given by $x_i = q_i^{-\varepsilon_i}$.
- ii. Assume that $q_i = 1 + t_i$ where t_i is the tax on good i . Employ the inverse elasticity rule to calculate the ratio t_1/t_2 of the optimal tax rates.
- iii. Find the optimal rates when revenue of 10 is to be collected.
- iv. Describe how your approach to the answer would be modified if indirect utility was

$$\text{determined by the function } V = a - \frac{q_1^{1-\varepsilon_1} q_2^{1-\varepsilon_2}}{w[1-\varepsilon_1][1-\varepsilon_2]} + \frac{1}{w}$$

7. There are quite a few ways to measure the efficiency cost (excess burden, dead weight loss, welfare cost) of distortionary taxation. A commonly used approach is to calculate the "equivalent variation" caused by a tax-induced price change and then compare it to the amount of tax revenue raised. The difference is the excess burden of the tax.

Consider an individual who has a "Cobb-Douglas" utility function of the form:

$$U(X, H) = X^{0.7} H^{0.3}$$

where X is consumption of a composite good and H is consumption of housing. This particular utility function implies that the individual always spends 70 percent of his income on the composite good and the remaining 30 percent on housing, regardless of the relative prices of the two goods; that is, the income and price elasticities of both goods are equal to one. (This feature of the individual utility function greatly simplifies the calculations required to answer the questions below.) The individual has an annual income of \$30,000. The price of the composite good is fixed at one, and the annual price (to producers) of housing is fixed at three dollars per square foot.

- A. Calculate the consumption of the composite good, the consumption of housing, and the price of housing in this initial no-tax equilibrium. What utility level (in "utils") is achieved by the individual in this equilibrium?
- B. Suppose that a 50 percent excise tax is placed on housing (the price to consumers increases to \$4.50 per square foot). Calculate the consumption of the composite good, the consumption of housing, and the price of housing in this new equilibrium. What utility level is achieved by the individual in this equilibrium? How much revenue is raised by the government?
- C. The "equivalent variation" caused by this tax-induced increase in the price of housing is defined as the amount of income the individual would be willing to pay to avoid

the tax-induced increase in the price of housing. Note that the definition of equivalent variation implies that prices should be viewed as being held constant at their initial levels. Calculate the equivalent variation of the 50 percent excise tax on housing. (Hint: The easiest way to do this is to use the “constant expenditure share” feature of the Cobb-Douglas utility function to determine individual demands for X and H as a function of the gross price of housing and income (the price of X is fixed at one), and substitute these demands into the formula for utility. This yields a formula for utility as a linear function of income when prices are constant. It is then straightforward to calculate the equivalent variation – the maximum amount of income the individual would be willing to pay in order to avoid the negative effects on utility of the tax increase.)

- D. In this case, the definition of the excess burden of the tax on housing is the difference between the equivalent variation and the amount of tax revenue raised. Calculate this excess burden. What is the excess burden per dollar of revenue raised?
- E. Try to approximate the excess burden of the tax on housing by assuming a linear demand curve, and calculating the standard “dead weight loss triangle” measure of excess burden – using the actual observed market demand curve rather than the appropriate “Hicksian” or “substitution effect only” demand curve. (The supply curve is linear by the assumption of a fixed producer price of housing.) Does this approach yield a reasonable approximation to the excess burden of the tax?
- F. Suppose the tax rate is reduced to 25 percent. Calculate the excess burden of the tax in this case. (Calculate both the absolute amount and the excess burden per dollar of tax revenue.) Are your results consistent with the rule of thumb regarding how the total excess burden of a tax varies with the tax rate?