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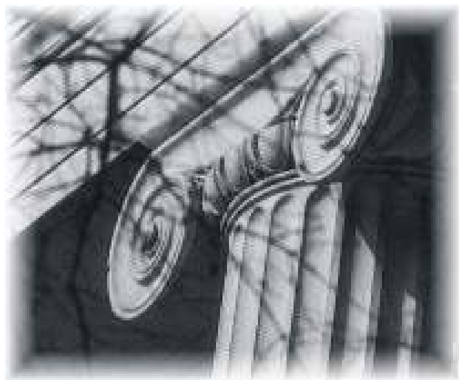

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OPTION PRICE WITHOUT EXPECTED UTILITY

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Abstract

The option price (OP) is commonly acknowledged as the preferred ex ante welfare measure, and is derived in the context of the expected utility (EU) model. We consider the meaning of this ex ante welfare measure in the rank-dependent expected utility (RDEU) framework, finding key differences with the EU-OP. The importance of this pertains to doing benefit-cost analysis when RDEU maximizers are prevalent in society.

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1. Introduction

In this manuscript, we consider the meaning of welfare measure under risk, allowing for the possibility that preferences do not correspond to all of the axioms of the expected utility model. As a specific alternative to the expected utility (EU) model we consider the rank-dependent EU (RDEU) model introduced by Quiggin (1982). By incorporating similar features into prospect theory (Kahneman and Tversky, 1979), Tversky and Kahneman (1992) develop cumulative prospect theory (CPT), which has become another widely used non-expected utility model in decision analysis under risk.

The option price, not to be confused with the price of financial options, is viewed as the desired measure of welfare when changes occur under conditions of known risk under collective risk[1] (Graham, 1981). Policy makers making decisions that fall under guidelines calling for an ex ante benefit-cost analysis ideally would estimate the option price for relevant changes. Relevant situations include policies affecting all federal lands or resources, and rule changes relating to some environmental regulations. An early example includes Desvousges *et al*. (1987), who estimated the option price for improved water quality used by recreational anglers. As a more recent example, Riddel and Shaw (2006) consider the potential loss in welfare (benefits estimated ex ante) from mortality risks tied to shipping nuclear wastes to the national high level nuclear waste repository at Yucca Mountain. Many other studies examine ex ante welfare measures for human health changes that are linked to deteriorations or improvements in air or water pollution (see the review in Shaw *et al*., 2005). The EU is assumed as the framework in almost all of the studies that include a formal expression for the welfare measure.

In laboratory experiments and other settings, many have come to view the EU axioms as restrictive. In particular, the independence axiom is often violated by individuals who are making choices under conditions of risk or uncertainty. This led to decades of research devoted to exploration of non-EU models, either by relaxing the independence axiom or another modification (see Starmer, 2000, for a survey of non-EU models). Modifications are typically accomplished by introducing non-linear probability weighting functions. While these alternatives have been greatly investigated, their

[1] For example, in a 2-state world with a collective risk, everyone will experience state 1 or everyone will experience state 2.

relationship to the option price has only been mentioned in a few, rare instances (see Smith 1992).

2. Option price under expected utility

We define a surplus in state i, s_i, as a compensating variation for the availability of a public good. Let u_i^0 denote a utility function in state i when the good is not available, and u_i denote a utility function in state i when the good is available. If the income in state i is w_i, the surplus in state i can be derived from $u_i(w_i - s_i) = u_i^0(w_i)$. If expected utility theory holds and there are two states of the world that occur with probabilities p and $1 - p$, the expected utility of an individual after paying the state-dependent surplus is

$$pu_1(w_1 - s_1) + (1-p)u_2(w_2 - s_2) \equiv \bar{u} . \quad (1)$$

Graham (1981) defines the *option price* (T) by an ex-ante willingness to pay for the good that keeps the expected utility unchanged, i.e.,

$$pu_1(w_1 - T) + (1-p)u_2(w_2 - T) = \bar{u} . \quad (2)$$

The option price can be greater or smaller than the expected surplus, $E(s) = ps_1 + (1-p)s_2$, for a risk-averse individual. Graham (1981) proposes a graphic approach to the option price by using the willingness-to-pay (WTP) locus, a depiction of possible pairs of willingness to pay, or contingent payment points. Let (x_1, x_2) be the vector of payments in states 1 and 2. The WTP locus (x_1, x_2) is constructed and developed such that

$$pu_1(w_1 - x_1) + (1-p)u_2(w_2 - x_2) = \bar{u} . \quad (3)$$

The WTP locus is smooth and concave because the agent is assumed to be a risk-averse EU maximizer. The option price can be found from the intersection of the WTP locus and the 45 degree line drawn through the origin (see Figure 1), as it indicates an equal pair of contingent payments. The arrow-headed line is the iso-expected-value line. Therefore the value on the x_1 axis where the arrow-headed line crosses the 45 degree line is $E(s)$. For both graphs in Figure 1, we assume that $w_1 = w_2 = w$, and $u_1'(w) < u_2'(w)$ for all w. In panel (a), $s_1 < s_2$, while in panel (b), $s_1 > s_2$.

(a) $s_1 < s_2$ (b) $s_1 > s_2$

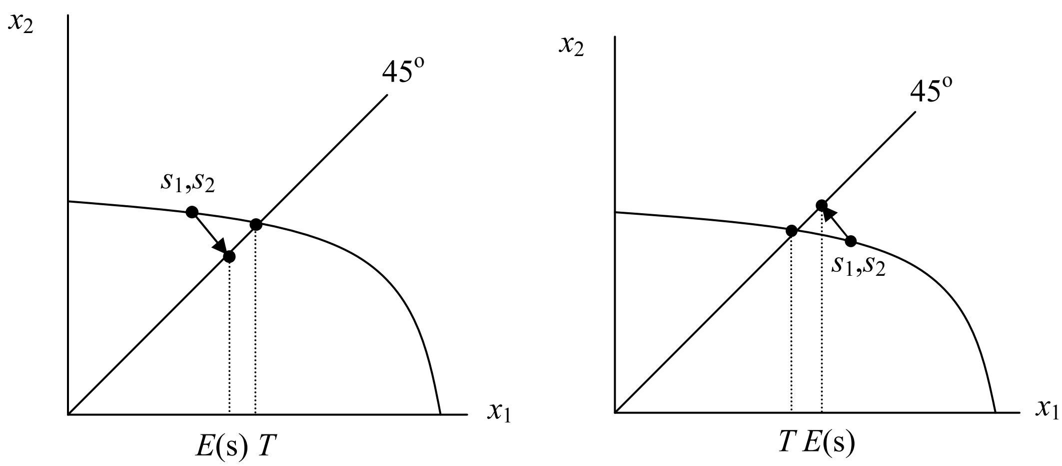

Figure 1: Willingness-to-pay loci under EU

3. The option price under rank-dependent expected utility

Now assume that there is a rank-dependent expected utility maximizer (see Quiggin, 1993, for key assumptions) whose utility function is the same as the EU agent. The essential features of the RDEU are that there is a non-linear weighting function h: $[0,1] \rightarrow [0,1]$ which is non-decreasing with $h(0) = 0$ and $h(1) = 1$, and that the RDEU maximizer ranks outcomes so that one outcome relative to another matters. This type of ranking is a feature in virtually all important alternatives to the EU, including CPT. To apply the RDEU features in a state-dependent utility framework, we follow Chiu's (1996) basic methodology. Chiu ranks prospective outcomes according to their state-dependent utility levels instead of their state-dependent income levels. Here assume that there are two states and state 2 is a preferred state, i.e., $u_1(w) < u_2(w)$ for all w, and $w_1 \leq w_2$. We assume further that after paying the surplus, state 2 is still preferred to state 1, i.e.,

$u_1(w_1 - s_1) < u_2(w_2 - s_2)$. If the utility function is state-dependent, the RDEU can be written as

$$h(p)u_1(w_1 - s_1) + (1 - h(p))u_2(w_2 - s_2) \equiv \bar{v}\,. \tag{4}$$

If $h(p) > p$, the individual overweights low utility outcomes, and the individual is said to be *pessimistic*. Pessimistic individuals dwell on the worst case scenario, attributing more importance to it than the true probability warrants. If $h(p) < p$, the individual overweights the high or best utility outcomes, and the individual is said to be *optimistic*. Applying Chiu's framework to Graham's option price concept, the RDEU option price (T') must satisfy the following equation:

$$h(p)u(w_1 - T') + (1 - h(p))u(w_2 - T') = \bar{v}\,. \tag{5}$$

We compare the option prices of two agents who have the same surplus vector; one is a state-dependent EU agent and another one is a state-dependent RDEU agent. Because society may well be mixed with people who are each type of agent, it is of interest to ask, which of the two types of agents has a higher option price?

Proposition 1. *Assume state-dependent utility and $s_1 < s_2$. If an EU agent and a pessimistic (an optimistic) agent have the same surplus in each state, then the pessimistic (optimistic) agent's option price is smaller (larger) than the EU agent's. If $s_1 > s_2$, then the pessimistic (optimistic) agent's option price is larger (smaller) than the EU agent's.*

Proof. Define $u(s_1, s_2) = pu_1(w_1 - s_1) + (1 - p)u_2(w_2 - s_2)$ and $v(s_1, s_2) = h(p)u_1(w_1 - s_1) + (1 - h(p))u_2(w_2 - s_2)$ with $h(p) > p$ so that u is utility for an EU agent and v is the same for a pessimistic agent. Since $u_1(w_1 - s_1) < u_2(w_2 - s_2)$ and $h(p) > p$, then $u(s_1, s_2) > v(s_1, s_2)$. Let T and T' be the option price of agent u and v, respectively. Then $u(s_1, s_2) = u(T, T)$ and $v(s_1, s_2) = v(T', T') = \bar{v}$. We have the following equations:

$$u(s_1, s_2) - \bar{v} = [h(p) - p][u_2(w_2 - s_2) - u_1(w_1 - s_1)] \quad (6)$$

$$u(T', T') - \bar{v} = [h(p) - p][u_2(w_2 - T') - u_1(w_1 - T')] \quad (7)$$

Let c be a function mapping from payment in state 1 (x_1) to payment in state 2 (x_2) such that $v(x_1, c(x_1)) = \bar{v}$. It follows that $c' < 0$. Since $v(s_1, s_2) = v(T', T')$ and $s_2 > s_1$, then $s_2 > T' > s_1$. It follows that $u_2(w_2 - T') > u_2(w_2 - s_2)$ and $u_1(w_1 - s_1) > u_1(w_1 - T')$. Therefore, the right-hand side of (7) is greater than that of (6). Then $u(T', T') > u(s_1, s_2) = u(T, T)$, and $T > T'$. If the agent is optimistic, then $h(p) < p$ and consequently $T' > T$. If $s_1 > s_2$ the results are reversed. □

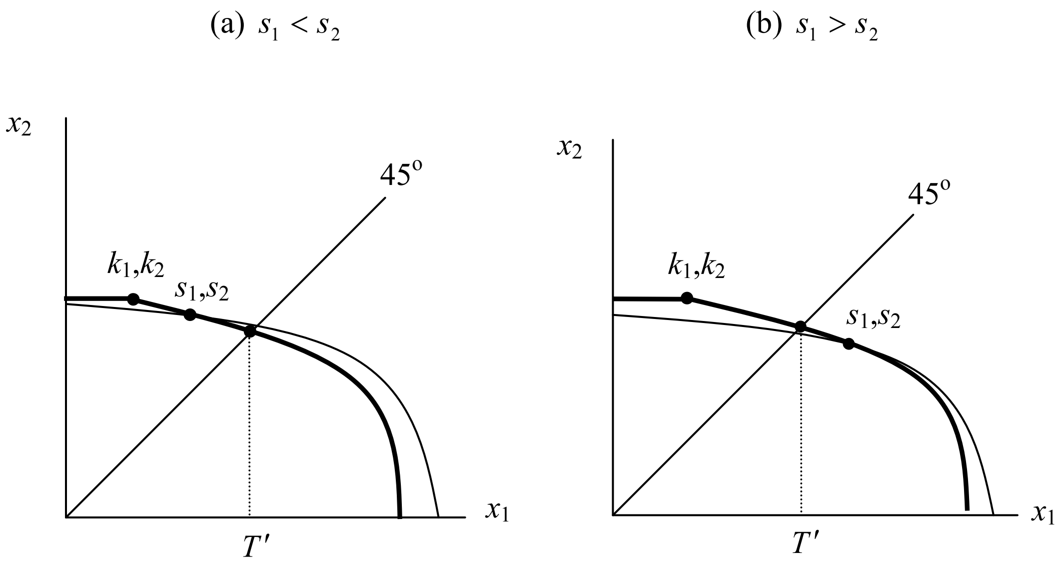

Figure 2: Willingness-to-pay loci under RDEU

The results in Proposition 1 can be illustrated in Figure 2. The bold curves are the WTP loci for pessimistic agents. In panel (a), $s_1 < s_2$ and the pessimistic agent's option price is lower than the EU agents's. In panel (b), $s_1 > s_2$ and the pessimistic agent's option price is higher than the EU agents's. Note that the WTP locus now has a kink at (k_1, k_2), defined as $u_1(w_1 - k_1) = u_2(w_2 - k_2)$. The WTP locus is not differentiable at

(k_1,k_2) when $h(p) \neq p$. This is similar to the kink on an indifference curve of an individual who is risk averse of order 1 defined by Segal and Spivak (1990)[2]. Segal and Spivak (1997) prove that the first-order risk aversion at (k_1,k_2) is equivalent to the local utility function that is not differentiable at (k_1,k_2).

4. Summary

Decision makers often make policy based on benefit-cost analysis, requiring examination of benefit, or welfare measures. Such welfare measures, in the presence of well-defined risks and when the expected utility (EU) framework appropriately models behavior, are possible to identify. For many reasons, some individuals behave in a manner that is inconsistent with the axioms of the EU. In such cases, a benefit-cost analysis should be based on welfare estimates that relate to their behavior and the more appropriate, non-EU framework. In this paper, we consider the meaning of an ex ante welfare measure in the rank-dependent expected utility (RDEU) framework, finding key differences. The importance of this pertains to doing benefit-cost analysis when RDEU maximizers are prevalent in society.

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[2] For a random variable ε with $E(\varepsilon) = 0$. Let $\mu(t\varepsilon)$ be the risk premium that the agent is willing to pay to avoid the risk $t\varepsilon$. The agent is risk averse of order 1 if $\partial\mu(t\varepsilon)/\partial t\,|_{t=0} \neq 0$.

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