Unanimous Inequality Rankings
and
Normalized Stochastic Dominance

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Abstract

A more general method for obtaining unanimous inequality rankings is derived. Normalized stochastic dominance is shown to imply unanimity for all inequality measures satisfying both the Pigou-Dalton principle of transfers and the transfer sensitivity axiom. The new procedure can be applied to distributions with unequal means and at higher degrees of stochastic dominance. Second-order normalized stochastic dominance is equivalent to Lorenz dominance. Third-order normalized stochastic dominance provides an operational procedure for making unanimous inequality comparisons when Lorenz curves cross. The large sample properties of normalized stochastic dominance are derived and an appropriate statistical inference test is proposed.

Key Words: Inequality Rankings, Unanimity, Normalized Stochastic Dominance

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I. Introduction

Atkinson (1970) demonstrated a basic principle of unanimous inequality rankings and established Lorenz dominance as a robust criterion for measuring relative income inequality. When Lorenz dominance exists, all summary measures of inequality satisfying the Pigou-Dalton principle of transfers yield the same ordinal inequality ranking. If two Lorenz curves cross, one can always find an index that ranks each of the underlying distributions as being more equal. Beach and Davidson (1983) developed a distribution-free statistical inference procedure which greatly enhances the power of Lorenz dominance comparisons in empirical research. However, even when combined with powerful statistical inference procedures Lorenz dominance remains incomplete in the sense that all pairs of distributions cannot generally be ranked.

When Lorenz curves intersect, unanimous inequality rankings can be extended by imposing axioms that are more restrictive than the Pigou-Dalton principle of transfers. The most appealing axiom for making such an extension is referred to as the "principle of diminishing transfers" (Kolm, 1976) and "transfer sensitivity" (Shorrocks and Foster, 1987). This axiom requires that the inequality reduction due to a progressive income transfer be inversely related to the level of income. Shorrocks and Foster (1987) extend unanimous inequality rankings by showing that, for distributions with equal means, third degree stochastic dominance implies that *all* inequality measures satisfying both the principle of transfers and the transfer sensitivity axiom yield equivalent ordinal rankings of inequality. More recently, Davies and Hoy (1994) reexamine Lorenz crossings and provide a unanimous-ranking condition for all measures of this narrowed class that can be applied to distributions with unequal means.

Since the income distributions of interest almost always have unequal means, Davies and Hoy's (1994) procedure meaningfully extends unanimous inequality rankings. However, for several reasons the procedure is less than ideal.[1] In this paper we provide a fresh and more general approach to unanimous inequality rankings, which is referred to as "normalized stochastic dominance." We build upon the close link between inequality orderings and stochastic dominance first established by Atkinson (1970) and extended by Shorrocks and Foster (1987) and generalize the relation between dominance and unanimity to distributions with unequal means and to higher degrees of stochastic dominance.

We show that second-order normalized stochastic dominance is equivalent to Lorenz and generalized Lorenz dominance (Shorrocks,

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1. There are several reasons for this. Davies and Hoy's method does not rest upon either Lorenz or stochastic dominance and, as a result, lacks the intuitive appeal of these ranking procedures. Further, the Davies and Hoy method cannot be extended to higher degrees of stochastic dominance and, as a consequence, can never yield a complete and unanimous inequality ordering. In addition, the Davies and Hoy criterion does not incorporate statistical inference procedures. Finally, empirically applying the Davies and Hoy procedure is tedious and it is not clear that it will prove to be empirically operational. In this regard we note that the procedure requires exact knowledge of the number of times the Lorenz curves cross and the conditional coefficient of variation at each crossing point. As a practical matter, income distribution survey data must be used and it is impossible to determine with precision the true number of Lorenz crossings in the entire populations of interest.

1983), but is a distinct ranking criterion. A major advantage of normalized stochastic dominance is that it can be extended to higher degrees of stochastic dominance and to distributions with unequal means. We demonstrate that third-order normalized stochastic dominance implies unanimous inequality rankings that are consistent with both the principle of transfers and the transfer sensitivity axiom. In principle, normalized stochastic dominance can be extended beyond the third degree to still higher orders of dominance and used to derive unanimous inequality rankings.[2]

The remainder of the paper is organized as follows. Section 2 briefly discusses important developments regarding unanimous inequality rankings. Section 2 also defines normalized stochastic dominance and demonstrates the main results. Section 3 derives the large sample properties of normalized stochastic dominance, which insures that the new ranking criterion is empirically operational. The final section provides concluding remarks.

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2. The difficulty with extending the normalized dominance approach to derive unanimous inequality rankings at the fourth and higher orders of stochastic dominance is that, to date, there have been no inequality axioms have been proposed that provide an intuitively appealing rationale for doing so. Given the results reported below, the computational and statistical inference procedures for making such extensions are straightforward. However, without an axiomatic foundation, unanimity rankings based upon fourth and higher-order dominance lack appeal. To overcome this problem we suggest an appealing axiom below (see fn. 7), which may establish unanimity for all relative measures of inequality that are ranked by fourth-order normalized stochastic dominance.

II. Lorenz Dominance, Stochastic Dominance and Normalized Stochastic Dominance

Consider two distributions f and g drawn from income space Ω over x and $x \in [0,\infty)$. We assume that f and g have continuous cumulative distribution functions $F(x)$ and $G(x)$ and denote their mean incomes as μ_f and μ_g.

The Lorenz curve ordinate for a given p, $p \in [0,1]$, is the income share of the $100p\%$ lowest income recipients, that is,

$$(1) \qquad L(f;p) = \frac{1}{\mu_f} \int_0^{F^{-1}(p)} x dF(x)$$

where $F^{-1}(p)$ denotes the income level corresponding to p. A distribution f Lorenz dominates g, or, $f\mathbf{L}g$ if $L(f;p) \geq L(g;p)$ for all $p \in [0,1]$. If we denote $F_2(x) = \int_0^x F(s)ds$ and generally $F_k(x) = \int_0^x F_{k-1}(s)dF(s)$ for $k \geq 3$, then distribution f displays *second order stochastic dominance over* g, or f $\mathbf{D}_2$ g, if $F_2(x) \leq G_2(x)$ for all $x \in [0,\infty)$ and $F_2(x) < G_2(x)$ for some $x \in [0,\infty)$; distribution f displays *kth-order stochastic dominance* g, or f $\mathbf{D}_K$ g, if $F_k(x) \leq G_k(x)$ for all $x \in [0,\infty)$ and $F_k(x) < G_k(x)$ for some $x \in [0,\infty)$.

An *inequality measure* is a continuous function I: $\Omega \rightarrow [0,\infty)$ whose value $I(f)$ indicates the inequality value associated with distribution f. An inequality measure is *relative* if it is independent of the mean. For ease of reference, we define two classes of inequality measures: C_2 and C_3. Class C_2 includes all *relative* inequality measures satisfying the Pigou-Dalton principle of transfers -- the transfer of income from a higher income person to a lower income person reduces inequality. Class C_3 consists of all *relative* inequality measures satisfying the Pigou-Dalton principle of transfers and the transfer sensitivity axiom -- the reduction in the inequality due to a progressive income transfer is inversely related to the income level. Obviously, $C_2 \supset C_3$.

We now state some well-known results due to Atkinson (1970) and Shorrocks and Foster (1987):

Proposition 1 For two distributions f and g, the following three statements are equivalent:

(*i*) $f \mathbf{L} g$

(*ii*) $I(f) < I(g)$ for all $I \in C_2$, and

(*iii*) $f \mathbf{D}_2 g$ if $\mu_f = \mu_g$.

When Lorenz curves cross, Davies and Hoy (1994) establish a condition for a unanimous inequality verdict by measures in C_3. Suppose the Lorenz curve for f crosses (initially from above) the Lorenz curve of g q times at p_i (i = 1, 2,..., q) with $0 < p_1 < p_2 < \ldots < p_q < 1$. Also denote $V_i(f)$ as the conditional coefficient of variation of the sub-population of f for incomes below $F^{-1}(p_i)$. With this we can state the following result which is a combination of Shorrocks and Foster (1987) and Davies and Hoy (1993).

Proposition 2 For two distributions f and g, the following statements are equivalent:

(i) $V_i(f) \leq V_i(g)$ for $i = 1, 2, \ldots, q+1$ with $p_{q+1} = 1$,[3]
(ii) $I(f) < I(g)$ for all $I \in C_3$, and
(iii) $f \mathbf{D}_3 g$ if $\mu_f = \mu_g$.

Although Davies and Hoy (1993) provide a method for checking for unanimous inequality rankings when Lorenz dominance fails, the approach does not yield a dominance criterion of the Lorenz or stochastic dominance type. When the means of two distributions are equal, one does not need to check Condition (*i*) and Shorrocks and Foster's (1987) result on third-order stochastic dominance provides a superior procedure for establishing unanimity. However, when the means of the distributions of interest are unequal, the ordinary third-order dominance criteria (*iii*) is no longer applicable. It is clear from the above that it would be very useful to have a new dominance criterion, which is a hybrid of Lorenz and second and third-order stochastic dominance.[4] Like Lorenz dominance, such a new criterion should be mean independent; and like Shorrocks and Foster's procedure, should provide unanimous inequality rankings. We provide this type of hybrid dominance criterion and refer to it below as "normalized stochastic dominance".

To establish the new dominance criterion for unanimous inequality rankings, we define normalized stochastic dominance as

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3. Note that this result is valid only for all relative inequality measures. However, Davies and Hoy (1994) do not specify this requirement.

4. Foster (1985) seems to recognize the need for a criterion of this type, but notes (1985, p.54), "It is an open question whether a more intuitive criterion using the Lorenz curve or some other construct might exist."

follows.

Definition. For a distribution f, its *normalized distribution*, $\tilde{f}$, is defined as the distribution of the normalized incomes, $y = x/\mu_f$, hence $\mu_{\tilde{f}} = 1$. The corresponding *c.d.f.* is denoted as $\tilde{F}(y)$ with $y \in [0,\infty)$. Then distribution f displays *second-order normalized stochastic dominance* over g, or f $\mathbf{ND}_2$ g, if $\tilde{F}_2(y) \leq \tilde{G}_2(y)$ for all $y \in [0,\infty)$ and $\tilde{F}_2(y) < \tilde{G}_2(y)$ for some $y \in [0,\infty)$; generally distribution f displays *kth-order normalized stochastic dominance* g, or f $\mathbf{ND}_k$ g, if $\tilde{F}_k(y) \leq \tilde{G}_k(y)$ for all $y \in [0,\infty)$ and $\tilde{F}_k(y) < \tilde{G}_k(y)$ for some $x \in [0,\infty)$.

It can be seen immediately that for any relative inequality measure I, $I(\tilde{f}) = I(f)$, because the difference between f and $\tilde{f}$ is the scale and a relative inequality is mean-independent and hence measure I will rank f and $\tilde{f}$ indifferently in terms of inequality level. With this observation and Propositions 1 and 2 we may present the following result.

Proposition 3. For k = 2 and 3, the following two statements are equivalent.

(*i*) $I(f) < I(g)$ for all $I \in C_k$,

(*ii*) f $\mathbf{ND}_k$ g.

Proposition 3 says that second and third-order normalized stochastic dominance relationships are necessary and sufficient conditions for unanimous inequality ranking by all measures in Class C_2 and Class C_3 respectively. Hence, to make comparisons between any pair of distributions, one needs to check only whether a normalized stochastic dominance relationship exists. Normalized stochastic dominance technique can be applied in the same straightforward fashion as Lorenz dominance.

By noting that $\tilde{F}(y)=F(y\mu_f)$ we can express Proposition 3 in the following equivalent form:

Proposition 3'. For k = 2 and 3, the following two statements are equivalent.

(*i*) $I(f) < I(g)$ for all $I \in C_k$,

(*ii*) $\frac{F_k(y\mu_f)}{\mu_f^{k-1}} \leq \frac{G_k(y\mu_g)}{\mu_g^{k-1}}$ for all $y \in [0,\infty)$ with strict

inequality holding for some $y \in [0,\infty)$.

This result suggests that to make inequality comparisons when the means of the distributions are unequal we should not compare $F_k(x)$ with $G_k(x)$ for all incomes x; instead, the appropriate procedure is to compare $F_k(x)$ with $G_k(\alpha x)$, which is divided by a factor α^{k-1} with $\alpha = \mu_g/\mu_f$. Clearly, when $\mu_f = \mu_g$, normalized stochastic dominance and stochastic dominance are equivalent.

III. Large Sample Properties of Normalized Stochastic Dominance

To insure that normalized stochastic dominance is an empirically operational method for making unanimous inequality comparisons using income distribution micro data, a statistical inference test is required. This section derives the large sample properties for $\tilde{F}_k(y)$ and provides a simple distribution-free testing procedure for normalized stochastic dominance approach. Before stating our results it is useful to note an equivalence between $F_k(x)$ and the lower partial moments as follows

(2) $$\int_0^x (x-s)^{k-1} dF(s) = (k-1)!\, F_k(x),$$

for $k \geq 1$.[5] If we denote $T_k(f;x) = \int_0^x (x-s)^{k-1} dF(s)$ then the testing of kth-order stochastic dominance between f and g (i.e. comparison between $F_k(x)$ and $G_k(x)$ for all x) is equivalent to the comparison between $T_k(f;x)$ and $T_k(g;x)$ for all x. Hence, we need only to derive the large sample properties of the estimate of $T_k(f;x)$ for a given x.

Assume a random sample of size n, $x_1, x_2, \ldots x_n$ is independently drawn from a population with continuous distribution $F(x)$. Then for a given y, $T_k(\tilde{f};y)$ can be consistently estimated as follows

(3) $$\hat{T}_k(\tilde{f};y) = \frac{1}{n}\sum_{i=1}^{n}\left(y - \frac{x_i}{\overline{x}}\right)^{k-1} I(x_i \leq y\overline{x}),$$

where $\overline{x}$ is the sample mean, $I(x_i \leq y\overline{x})$ is one if $x_i \leq y\overline{x}$ and is zero otherwise. Since for a given y, $\hat{T}_k(\tilde{f};y)$ is a function of x_i and $\overline{x}$ then the following lemma which is due to Proschan and Pyke (1964) on the asymptotic distribution of $\frac{1}{n}\sum_{i=1}^{n} h(x_i,\overline{x})$ is very useful.

Lemma 1. If x_i and $h(x_i, \overline{x})$ have finite variances, σ_x^2 and σ_h^2, and covariance σ_{xh}, then

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5. This is a standard expression in the stochastic dominance literature, see Fishburn (1976, 1980). Foster and Shorrocks (1988) use this expression in proving their poverty ordering results.

(4) $$Z \equiv n^{1/2}\sum_{i=1}^{n}[h(x_i,\overline{x}) - E\{h(x_i,\mu_f)\}]$$

converges in distribution to a normal random variable with mean zero and variance $\sigma_h^2 + c^2\sigma_x^2 + 2c\sigma_{xh}$, where $c = E\{\frac{\partial h(x_i,\mu_f)}{\partial \mu_f}\}$.

By choosing $h(x_i,\overline{x}) = \left(y - \frac{x_i}{\overline{x}}\right)^{k-1} I(x_i \le y\overline{x})$ then it is easy to check that $h(x_i,\overline{x})$ satisfies all necessary conditions considered by Proschan and Pyke (1964) and the application of Lemma 1 leads immediately to the following result:[6]

Proposition 4. Under the conditions of Lemma 1,

(5) $$n^{1/2}\{\hat{T}(\tilde{f};y) - T(\tilde{f};y)\}$$

converges in distribution to a normal random variable with mean zero and variance $\omega_f = \sigma_h^2 + c^2\sigma_x^2 + 2c\sigma_{xh}$, where

$$\sigma_h^2 = \frac{1}{\mu_f^{2(k-1)}}\int_0^{y\mu_f}(y\mu_f - s)^{2(k-1)}dF(s) - \frac{1}{\mu_f^{2(k-1)}}\left(\int_0^{y\mu_f}(y\mu_f - s)^{k-1}dF(s)\right)^2,$$

(6) $$c = \frac{(k-1)}{\mu_f^k}\int_0^{y\mu_f}s(y\mu_f - s)^{k-2}dF(s), \text{ and}$$

$$\sigma_{xh} = \frac{1}{\mu_f^{k-1}}\int_0^{y\mu_f}s(y\mu_f - s)^{k-1}dF(s).$$

Proposition 4 is valid regardless of the underlying distribution F. The variance ω can be consistently estimated from the sample. Hence based on proposition 4 we can construct a

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6. Note that when $k = 2$, $\frac{\partial h(x_i,\mu_f)}{\partial \mu_f}$ is not continuous at $x_i = y\mu_f$. However, as pointed out by Proschan and Pyke, this discontinuity does not affect the validity of the proposition.

distribution-free testing procedure for normalized stochastic dominance.

For a pair of distributions f and g, we wish to distinguish the following four possibilities: f $\mathbf{ND}_k$ g; g $\mathbf{ND}_k$ f; $\tilde{f}$ *and* $\tilde{g}$ are identical; and $\tilde{f}$ kth-order stochastic crosses with $\tilde{g}$. Following Beach and Richmond (1985) we consider a joint test using critical values from the Studentized Maximum Modulus (SMM) distribution in the following procedure. We may first select y_j ($j = 1, 2, \ldots, M$) which spread over interval $[0,\infty)$ with $y_1 < y_2 \ldots < y_M$. Then we calculate M pairs of ordinates $\{\hat{T}_k(\tilde{f};y_j),\hat{T}_k(\tilde{g};y_j)\}$ and compare two estimates of each ordinate. If two samples of sizes n_f and n_g are independently drawn from populations with distributions $F(x)$ and $G(x)$, then the appropriate standard normal test statistic for $H_0:T_k(\tilde{f};y_j)=T_k(\tilde{g};y_j)$ against $H_{a1}:T_k(\tilde{f};y_j)>T_k(\tilde{g};y_j)$ or $H_{a2}:T_k(\tilde{f};y_j)<T_k(\tilde{g};y_j)$ is

(7) $$z_j = [\hat{T}_k(\tilde{f};y_j)-\hat{T}_k(\tilde{g};y_j)]/[(\hat{\omega}_f / n_f)+(\hat{\omega}_g / n_g)]^{1/2}$$

where $\hat{\omega}_f$ and $\hat{\omega}_g$ are the corresponding estimated variances.

The procedure proposed by Beach and Richmond (1985) allows a multiple comparison test for a set of differences by comparing z-statistics to appropriate critical values in the SMM table. Accordingly, one may perform a multiple comparison test on M pairs of ordinates and the joint test may reveal four possibilities: (a) f kth-order normalized stochastic dominates g if no significant positive z-value and at least one z-value is negative and significant; (b) g kth-order normalized stochastic dominates f if no significant negative z-value and at least one z-value is positive and significant; (c) $\tilde{f}$ *and* $\tilde{g}$ are identical if no significant (positive or negative) z-value; and (d) f kth-order normalized stochastic crosses with g if there are at least one positive significant z-value and at least one negative significant z-value.

The above test procedure can be applied to both second and third-order normalized stochastic dominance. For second-order applications set $k = 2$ and for third-order comparisons set $k = 3$. In fact the approach can be applied to any higher degree of stochastic dominance. It one wishes to find conditions for inequality ranking when both second and third order normalized stochastic dominance fail, then this approach can be utilized by choosing a larger value of k.[7]

IV. Concluding Remarks and a Comment on Lorenz versus Second-Order Normalized Stochastic Dominance

The concept of normalized stochastic dominance enables us to view income inequality from a fresh perspective, which leads to a

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7. It was noted above (fn. 2) that in the absence of an axiomatic foundation extensions of normalized stochastic dominance to the fourth and higher orders of dominance lack appeal. However, it is possible to introduce more restrictive axioms to characterize higher orders of normalized stochastic dominance. For example, using Shorrocks and Foster's (1987, 486) concept of favorable composite transfer, we suggest the following axiom, which we refer to as the fourth-order sensitivity axiom: inequality reduction due to a *favorable composite transfer* should be inversely related to the income level. We conjecture that this axiom may characterize fourth-order normalized stochastic dominance. A complete characterization of the axiom and its relation to fourth-order dominance would be useful, but is beyond the scope of the present paper. We point out that if one is willing to accept the transfer sensitivity axiom and inequality cannot be ranked by third-order normalized dominance, then it seems one should also be willing to accept the fourth-order sensitivity axiom.

new method for deriving unanimous inequality rankings. Like Lorenz dominance, second-order normalized stochastic dominance unanimously ranks all measures satisfying the Pigou-Dalton principle of transfers. However, unlike the Lorenz method,our normalized procedure can be extended to higher-orders of stochastic dominance and can be applied to distributions with unequal means. As a result, normalized stochastic dominance can be used to unanimously rank inequality when the Lorenz criterion fails.

We show that third-order normalized stochastic dominance implies unanimous inequality rankings for all measures satisfying both the transfer principle and transfer sensitivity axioms. Unlike Davies and Hoy's (1994) method, which also unanimously ranks inequality measures consistent with the principle of transfers and transfer sensitivity axiom, normalized stochastic dominance does not require knowledge of the exact number of times Lorenz curves intersect nor information on the conditional coefficients of variation up to each crossing point. Instead, normalized stochastic dominance can be applied in a straightforward fashion using well-developed and intuitively appealing dominance ranking techniques. Further, the new ranking criterion readily lends itself to statistical inference testing for stochastic dominance at the second, third or higher orders.

If third-order normalized dominance does not hold, then we can always find measures satisfying the transfer sensitivity axiom that ranks each of the distributions of interest as more unequally distributed. In this case normalized stochastic can be extended to fourth and higher orders of dominance. In principle, normalized stochastic dominance can be extended to obtain a unanimous and virtualy complete ranking. Thus, normalizing the distributions of interest by their means enables us to provide a dominance criterion that is a hybrid of Lorenz and ordinary stochastic dominance, which can be used to derive unanimous rankings in *all* pairwise comparisons of income distributions.

The results presented in this paper have immediate implications for measuring inequality when Lorenz curves intersect and the means of the distributions of interest are not equal. But we emphasize that the method is quite general and provides an alternative to Lorenz dominance. Further, the alternative is appealing because it can be extended to higher degrees of stochastic dominance and applied to distributions that have unequal means.

It may seem that the equivalence of the unanimity rankings derived using Lorenz and second-order normalized stochastic means that our normalized approach has nothing to offer when Lorenz curves do not cross. To be sure, the two criteria are theoretically equivalent as demonstrated by Propositions 1 and 3 above. However, for the reasons advanced below a conclusion that second-order normalized stochastic dominance has nothing new to offer is premature.

In empirical studies of unanimous inequality rankings Lorenz curves are often found to be "statistically equivalent" [c.f., Bishop, Formby and Smith (1991)], which means the null hypothesis of no difference cannot be rejected at conventional levels of significance. These results do not necessarily carryover to second-order normalized stochastic dominance. The variance-covariance dispersion matrix underlying the Beach-Davidson (1983) test for Lorenz dominance and the comparable matrix underlying normalized stochastic dominance are radically different; the former variance-covariance matrix is far more complex. Further, the Lorenz approach is based upon order statistics and proceeds by focusing on population quantiles (e.g. deciles); whereas normalized stochastic dominance *is not* based upon order statistics and statistical tests are conducted at selected income targets in the distribution. As a consequence, the two statistical ranking procedures may be quite different in terms of their ability to statistically order income distributions.

Many of the empirical Lorenz curve comparisons that are ranked as "equivalent" by the Beach-Davidson test for Lorenz dominance may be classified as a "crossing" or "dominance" by second-order normalized stochastic dominance. If future research reveals this to be the case, the explanation will lie in the statistical power of the tests. Determining the relative power of the Lorenz and normalized stochastic dominance inference tests is a complex research question, which is beyond the scope of the current paper.

We point to one additional advantage of normalized second-order stochastic dominance. As noted above, inference based Lorenz dominance is necessarily based upon order statistics and statistical tests can only be applied to independent income distribution samples. In contrast, inference based normalized stochastic dominance *does not require* independent samples. There are numerous inequality issues and questions involving dependent samples that cannot be addressed using inference based Lorenz dominance. For example, the redistributive effects of taxes and transfers, the impact of the choice of the equivalent scale on unanimous inequality rankings and the effects of working wives on relative income inequality all involve dependent samples. In general, inference based normalized stochastic dominance can be applied anytime marginal changes in income distributions occur and the impact on relative inequality is of interest. In a manner similar to Bishop, Chow and Formby (1994), second-order normalized stochastic dominance can be readily utilized to statistically test for marginal changes in relative inequality when the samples are dependent.

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